

# String Constraints on Discrete Symmetries in MSSM Type II Quivers

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Pascal Anastasopoulos<sup>1\*</sup>, Mirjam Cvetič<sup>2,3†</sup>, Robert Richter<sup>4‡</sup>, Patrick K.S. Vaudrevange<sup>5§</sup>

<sup>1</sup> *Technische Univ. Wien Inst. für Theoretische Physik, A-1040 Vienna, Austria*

<sup>2</sup> *Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104-6396, USA*

<sup>3</sup> *Center for Applied Mathematics and Theoretical Physics, University of Maribor, Maribor, Slovenia*

<sup>4</sup> *II. Institut für Theoretische Physik, Hamburg University, Germany*

<sup>5</sup> *Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, 22607 Hamburg, Germany*

**ABSTRACT:** We study the presence of discrete gauge symmetries in D-brane semi-realistic compactifications. After establishing the constraints on the transformation behaviour of the chiral matter for the presence of a discrete gauge symmetry we perform a systematic search for discrete gauge symmetries within local semi-realistic D-brane realizations, based on four D-brane stacks, of the MSSM and the MSSM with three right-handed neutrinos. The systematic search reveals that Proton hexality, a discrete symmetry which ensures the absence of R-parity violating terms as well as the absence of dangerous dimension 5 proton decay operators, is only rarely realized. Moreover, none of the semi-realistic local D-brane configurations exhibit any family dependent discrete gauge symmetry.

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\*pascal@hep.itp.tuwien.ac.at

†cvetic@cvetic.hep.upenn.edu

‡robert.richter@desy.de

§patrick.vaudrevange@desy.de

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## 1. Introduction

While the Minimal Supersymmetric Standard Model (MSSM) addresses various short-comings of the Standard Model (SM), such as solving the hierarchy problem, providing a natural dark matter candidate (i.e. the lightest supersymmetric particle, LSP) and gauge coupling unification, it exhibits some severe phenomenological problems, among them the issue of proton stability. The SM guarantees proton stability, whereas the MSSM allows renormalizable R-parity breaking operators consistent with supersymmetry and gauge invariance of the superpotential that do lead to a disastrous high proton decay rate.

The renormalizable SM gauge invariant superpotential terms read

$$W_{MSSM} = Y_U Q_L U_R H_u + Y_D Q_L D_R H_d + Y_L L E_R H_d + \mu H_u H_d \\ + \lambda_1 U_R D_R D_R + \lambda_2 Q_L L D_R + \lambda_3 L L E_R + \alpha L H_u , \quad (1.1)$$

where the terms in the first line are the Yukawa couplings giving mass to quarks and leptons after electroweak symmetry breaking as well as the  $\mu$ -term. On the other hand, the second line contains terms that do not conserve baryon and lepton number, so called R-parity violating terms. They can lead to rapid proton decay, rendering the LSP unstable and thus eliminating the possibility of any SUSY particle being the dark matter candidate. Moreover, SM gauge invariance allows also for the dimension 5 proton decay operators

$$Q_L Q_L Q_L L \quad U_R U_R D_R E_R , \quad (1.2)$$

which if not suppressed lead to a disastrous high proton decay rate.

Generically, discrete symmetries such as R-parity or Baryon triality are invoked to forbid the presence of those superpotential terms. Despite the fact that those discrete symmetries ensure the absence of such undesired terms their origin remains unclear. There exist strong arguments implying that in a consistent quantum gravity global symmetries, continuous or discrete, are broken by quantum gravity corrections [1–5]. An exception are discrete symmetries that have a gauge symmetric origin, so called discrete gauge symmetries. For instance abelian discrete symmetries  $\mathbf{Z}_N$  are remnants of continuous  $U(1)$  symmetries that are broken by scalars with charge  $N$  under the respective  $U(1)$  acquiring vev's. However, the presence of a discrete symmetry seems fine-tuned unless there is a dynamical reason for the scalar field with charge  $N$  to acquire an appropriate vev.

This might find an explanation within string theory. For example, as has been recently shown in [6], in type II compactifications discrete symmetries naturally appear as subgroups of anomalous  $U(1)$  gauge factors broken by Stückelberg type couplings. More concretely, D-brane compactifications exhibit multiple  $U(1)$ 's which generically appear anomalous whereas the anomalies are cancelled by the Green-Schwarz mechanism [7–15]. The  $U(1)$ 's become massive and are broken to a discrete abelian subgroup via the presence of a  $B \wedge F$  coupling, where  $B$  denotes the Ramond-Ramond 2-form. In the low energy effective theory the massive  $U(1)$ 's survive as global symmetries that are preserved by all perturbative quantities while D-instantons can break them inducing perturbatively absent couplings [16–18]. On the other hand the discrete symmetry, the remnant of the  $U(1)$  after the Green-Schwarz mechanism, is respected by all perturbative and non-perturbative quantities [6, 19].

Inspired by the work of [6] we want to investigate the presence of discrete symmetries in (semi-)realistic D-brane models. In a series of publications [20–22] the authors analysed so-called D-brane quivers, i.e. local D-brane configurations, with

respect to their phenomenology. They performed a systematic search for local D-brane setups that exhibit (semi-)realistic features using the bottom-up approach. More concretely, they specified the chiral spectrum to be the MSSM or the MSSM with three right-handed neutrinos and imposed the presence of quark and lepton Yukawa couplings on perturbative or non-perturbative level and at the same time required, among other criteria, the absence of R-parity violating terms as well as the absence of dimension 5 proton decay operators. They found of the order of 40 local D-brane configurations based on four D-brane stacks that are consistent with the global consistency conditions and exhibit a (semi-)realistic phenomenology. Given those D-brane quivers it is interesting whether the absence of R-parity as well as dimension 5 proton decay operators is accidental or is originated from a discrete gauge symmetry.

We will study the constraints arising from string theory for the presence of a discrete gauge symmetry in D-brane models. As we will see those stringy constraints do contain the usual four-dimensional discrete anomaly constraints, however, pose additional constraints related to higher dimensional anomalies upon decompactification. Established those constraints we analyse the promising local D-brane configurations found in [20–22] with respect to discrete gauge symmetries. We find that depending on the hypercharge embedding matter parity appears quite frequently, forbidding the undesired R-parity violating terms. Only very few D-brane quivers allow for Proton hexality which ensures the absence R-parity violating terms as well as the dangerous dimension 5 operators.

This paper is organized as follows. In section 2 we review the findings of the systematic search for discrete symmetries in the MSSM, using four-dimensional discrete anomaly conditions. In section 3 we discuss the constraints on the transformation behaviour of chiral matter that arise from string consistency conditions. Moreover, we establish the conditions on the transformation behaviour of the matter fields for the presence of a discrete gauge symmetry in D-brane compactifications. In section 4 we impose the constraints for the presence of a discrete gauge symmetry, studied before, for a class of intriguing local D-brane configurations that exhibit a (semi-)realistic phenomenology. We analyse what type of discrete symmetries can appear as well as their phenomenological implications. In section 5 we present our conclusions. The appendix A contains the details of the systematic bottom-up search for local D-brane configurations that give rise to a (semi-)realistic phenomenology.

## 2. Discrete gauge symmetries in the MSSM from a field theory perspective

In this section we review the results of the work [23] where the authors search for all possible family independent (non-R) discrete gauge symmetries within the MSSM.

	$Q_L$	$U_R$	$D_R$	$L$	$E_R$	$N_R$	$H_u$	$H_d$
$A$	0	0	-1	-1	0	1	0	1
$L$	0	0	0	-1	1	1	0	0
$R$	0	-1	1	0	1	-1	1	-1

**Table 1:** The family independent generators of discrete  $\mathbf{Z}_N$  gauge symmetries in the MSSM.

They find a finite class of discrete gauge symmetries that satisfy the four-dimensional discrete gauge anomaly constraints, i.e. the mixed  $\mathcal{A}_{SU(3)SU(3)\mathbf{Z}_N}$ ,  $\mathcal{A}_{SU(2)SU(2)\mathbf{Z}_N}$  as well as the gravitational anomaly  $\mathcal{A}_{GG\mathbf{Z}_N}$ . In their search they furthermore require the family independent discrete gauge symmetries to allow for the Yukawa couplings

$$Q_L H_d D_R \quad Q_L H_u U_R \quad L H_d E_R . \quad (2.1)$$

As already shown in [24]<sup>1</sup> any family independent discrete gauge symmetry  $\mathbf{Z}_N$  of the MSSM with generator  $g_N$  can be expressed in terms of products of powers of three mutually commuting generators  $A_N$ ,  $L_N$  and  $R_N$ , i.e.

$$g_N = A_N^n \times L_N^p \times R_N^m , \quad (2.2)$$

where the exponents run over  $m, n, p = 0, 1, \dots, N-1$ . The charges of the chiral MSSM matter fields under these three independent  $\mathbf{Z}_N$  are displayed in table 1. Given this assignment the matter fields carry discrete charges

$$\begin{aligned} q_{Q_L} &= 0 & q_{U_R} &= -m & q_{D_R} &= m - n \\ q_L &= -n - p & q_{E_R} &= m + p & q_{H_u} &= m & q_{H_d} &= -m + n \end{aligned} \quad (2.3)$$

under a  $g_N$  transformation.

The discrete gauge anomaly constraints applying the charge assignment (2.3) read (see also [26])

$$SU(3) - SU(3) - \mathbf{Z}_N : \quad 3n = 0 \mod N \quad (2.4)$$

$$SU(2) - SU(2) - \mathbf{Z}_N : \quad 2n + 3p = 0 \mod N \quad (2.5)$$

$$G - G - \mathbf{Z}_N : \quad -13n - 3p + 3m = 0 \mod N + \eta \frac{N}{2} , \quad (2.6)$$

where the first two lines correspond to the discrete gauge anomalies of  $SU(3)$  and  $SU(2)$ , respectively. The last line describes the gravitational anomaly, where  $\eta = 0$  for  $N$  being odd and  $\eta = 1$  for  $N$  being even. The last term of (2.6) takes into account the possibility of heavy Majorana fermion fields.

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<sup>1</sup>See, also [25].

Solving these discrete gauge anomaly constraints one finds a finite class of solutions [23], ranging from  $\mathbf{Z}_2$  up to  $\mathbf{Z}_{18}$  symmetries. In table 2 all possible family independent discrete gauge symmetries of the MSSM are displayed in terms of the three  $\mathbf{Z}_N$  generators,  $A_N$ ,  $L_N$  and  $R_N$ .

$N$	$n$	$p$	$m$	Discrete gauge symmetries
2	0	0	1	$R_2$
3	0	0	1	$R_3$
	0	1	(0, 1, 2)	$L_3, L_3 R_3, L_3 R_3^2$
6	0	0	1	$R_6$
	0	2	(1, 3, 5)	$L_6^2 R_6, L_6^2 R_6^3, L_6^2 R_6^5$
9	3	1	(2, 5, 8)	$A_9^3 L_9 R_9^2, A_9^3 L_9 R_9^5, A_9^3 L_9 R_9^8$
	3	4	(2, 5, 8)	$A_9^3 L_9^4 R_9^2, A_9^3 L_9^4 R_9^5, A_9^3 L_9^4 R_9^8$
	3	7	(2, 5, 8)	$A_9^3 L_9^7 R_9^2, A_9^3 L_9^7 R_9^5, A_9^3 L_9^7 R_9^8$
18	6	2	(1, 7, 13)	$A_{18}^6 L_{18}^2 R_{18}, A_{18}^6 L_{18}^2 R_{18}^7, A_{18}^6 L_{18}^2 R_{18}^{13}$
	6	8	(1, 7, 13)	$A_{18}^6 L_{18}^8 R_{18}, A_{18}^6 L_{18}^8 R_{18}^7, A_{18}^6 L_{18}^8 R_{18}^{13}$
	6	14	(1, 7, 13)	$A_{18}^6 L_{18}^{14} R_{18}, A_{18}^6 L_{18}^{14} R_{18}^7, A_{18}^6 L_{18}^{14} R_{18}^{13}$

**Table 2:** All fundamental discrete gauge symmetries in the MSSM satisfying the anomaly cancellation conditions [23]. Here one allows for heavy fermions with fractional charges.

For the MSSM with 3 right-handed neutrinos only a subgroup of the discrete symmetries displayed in table 2 can be realized. Requiring the presence of the Dirac neutrino mass  $LH_u N_R$  implies the charge

$$q_{N_R} = n + p - m \quad (2.7)$$

under the discrete symmetry. Since the neutrinos are not charged under the  $SU(3)$  and  $SU(2)$  their presence will only lead to changes in the gravitational discrete gauge anomaly, which is then given by

$$G - G - \mathbf{Z}_N : \quad -10n = 0 \pmod{N + \eta \frac{N}{2}}, \quad (2.8)$$

which together with the other two discrete gauge anomaly constraints allows only for solutions with  $n = 0$ . Thus in contrast to the pure MSSM the MSSM with three additional right-handed neutrinos does not allow any  $\mathbf{Z}_9$  and  $\mathbf{Z}_{18}$  symmetries. On the other hand all  $\mathbf{Z}_2$ ,  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  symmetries displayed in table 2 are realized. Beyond those the MSSM with three right-handed neutrinos does not exhibit any further family independent discrete gauge symmetries.

For a given discrete gauge symmetry, i.e. for a specific choice of the parameters  $m$ ,  $n$  and  $p$ , we can determine with eq. (2.3) the discrete charges of the SM fields and study the appearances of various terms in the superpotential. Specifically it is interesting whether a discrete symmetry forbids some of the undesired couplings such

coupling	$R_2$	$L_3 R_3$	$R_3$	$L_3$	$L_3 R_3^2$	$L_6^2 R_6^5$	$R_6$	$L_6^2 R_6^3$	$L_6^2 R_6$	$\mathbf{Z}_9$ & $\mathbf{Z}_{18}$
$H_u H_d$	✓	✓	✓	✓	✓	✓	✓	✓	✓	
$L H_u$		✓								
$L L E_R$		✓								
$Q_L L D_R$		✓								
$U_R D_R D_R$				✓						
$Q_L Q_L Q_L L$	✓		✓				✓			
$U_R U_R D_R E_R$	✓		✓				✓			
$L H_u L H_u$	✓	✓				✓				
$N_R N_R$	✓	✓				✓				

**Table 3:** Allowed superpotential terms for the respective discrete gauge symmetries [23].

as R-parity violating terms or dangerous dimension 5 proton decay operators. Table 3 depicts for all possible family independent discrete gauge symmetries the allowed superpotential terms.

Let us discuss some of the intriguing discrete symmetries displayed in table 3. The  $\mathbf{Z}_2$  symmetry  $R_2$  is the usual matter parity [27] while  $L_3 R_3$  is Baryon triality [24]. Proton hexality, basically the product of matter parity and Baryon triality, is given by  $L_6^2 R_6^5$  and forbids all R-parity violating terms as well as the dangerous dimension 5 proton decay operators while still allowing for a  $\mu$ -term  $H_u H_d$  and the Weinberg operator  $L H_u L H_u$ .

The above discussion on the allowed couplings for the respective discrete gauge symmetry applies specifically to the MSSM. Allowing for additional singlets, such as right-handed neutrinos, which do not acquire any vev does not change the analysis. However, the presence of right-handed neutrinos accompanied with a Dirac neutrino mass term raises the issue of the generation of small neutrino masses. A particular intriguing mechanism is the see-saw mechanism that requires large Majorana mass terms for the right-handed neutrinos. In the last line of table 3 we display which of the discrete symmetries permits for a Majorana mass term and thus allows the generation of small neutrino masses via the see-saw mechanism.

Finally, there exist two additional classes of discrete gauge symmetries, namely non-abelian discrete gauge symmetries and discrete R-symmetries. As recently pointed out the latter may play a special role in GUT theories, realized as a  $\mathbf{Z}_4^R$  symmetry that forbids all R-parity violating terms as well as dimension 5 proton decay operators [28–30]. On the other hand non-abelian discrete gauge symmetries are often times invoked explaining various observations in flavour physics (see e.g. [31]). In this work we perform a systematic bottom-up D-brane analysis which ignores any specifics of the internal geometry. However non-abelian discrete gauge symmetries as well as discrete R-symmetries do rely on the details of the compactification manifold.

Thus here we focus only on the subset of abelian discrete gauge symmetries.

### 3. Discrete symmetries in D-brane compactifications

In this work we want to perform a systematic bottom-up study for discrete gauge symmetries within the class of realistic local D-brane configurations based on four D-brane stacks found in [20–22]<sup>2</sup>. There the authors studied local D-brane constructions, where the gauge degrees of freedom are given by open strings attached to a D-brane stack whereas the chiral matter appears at the intersection of two D-brane stacks. As we will review below the distribution of the chiral matter is not arbitrary but subject to severe constraints arising from string consistency constraints, such as tadpole cancellation.

In the work [20–22] the authors systematically analysed all four-stack configurations, four stack quivers, imposed the severe consistency constraints as well as required some (semi-) realistic features. More specifically, they demanded the chiral spectrum to be one of the MSSM or MSSM plus three right-handed neutrinos, required the presence of the quark and lepton Yukawa couplings, the absence of R-parity violating superpotential terms as well as the absence of dimension 5 proton decay operators. Moreover, in this search they asked for a mechanism that explains the small neutrino masses and demanded quark Yukawa textures that are in agreement with the CKM matrix. They found of the order of 40 local D-brane configurations allowing for an intriguing low energy phenomenology. Here we want to analyse those promising four stack quivers with respect to discrete gauge anomalies, i.e. we study whether the absence of undesired superpotential terms can be explained by the presence of a discrete gauge symmetry.

Recently discrete gauge symmetries attracted a lot of attention in the construction of realistic string theory model building. In the context of heterotic string theory, for specific toroidal compactifications one could identify proton hexality, forbidding for this specific model the presence of R-parity violating couplings and dimension 5 proton decay operators [45]. Moreover, in [46] the authors found for a similar construction a  $\mathbf{Z}_4^R$  symmetry realized that forbids any undesired couplings and allows the  $\mu$ -term only non-perturbatively, thus giving an explanation for the small value of around 100 GeV.

Inspired by the work of Banks and Seiberg [5] discrete gauge symmetries were studied also in the context of D-brane compactifications. In [6] (see also [19]) abelian discrete gauge symmetries arising from anomalous  $U(1)$  gauge factors were investigated. This study has been very recently extended to abelian and non-abelian discrete gauge symmetries arising from isometries of the compactification manifold [47].

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<sup>2</sup>The first local bottom-up constructions were discussed in [32–34]. For recent analogous work on semi-realistic bottom-up searches, see [35–44].

As discussed above we want to perform a systematic study of discrete gauge symmetries within a class of local D-brane configurations without making any reference to the details of the compactification manifold. Thus in this work we focus on the first class of abelian discrete gauge symmetries investigated in [6]. There the authors discuss the presence of abelian discrete gauge symmetries in D-brane compactifications which are remnants of anomalous  $U(1)$  gauge symmetries which generically appear in D-brane compactifications. Those anomalous  $U(1)$  gauge symmetries become massive via the Green-Schwarz mechanism and survive as global symmetries on the perturbative level. D-instanton effects can break those global symmetries inducing sometimes desired, but perturbatively forbidden, couplings, such as Majorana mass terms for the right-handed neutrinos [16, 17, 48, 49] or particular Yukawa couplings in GUT theories [50]. As shown in [6] discrete abelian gauge symmetries in D-brane compactifications are not broken by non-perturbative effects and thus hold not only at all levels in perturbation theory, but also at the non-perturbative level.

Here we focus on the concrete case of type IIA constructions with intersecting D6 branes, but an analogous discussion applies to the T-dual type IIB picture with D-branes on singularities as well as the type I compactification with magnetized D9 branes. In those compactifications D6-branes fill out the four-dimensional space-time and wrap three-cycles  $\pi_x$  in the internal manifold<sup>3</sup>. A stack of  $N$  D6-branes gives rise to an  $U(N)$  gauge theory, that splits into  $U(N) = SU(N) \times U(1)$  where the abelian part is generically anomalous. It becomes massive via the Stückelberg mechanism and does not appear in the low-energy field theory dynamics. In [33] the authors give the criteria for the existence of an unbroken abelian gauge symmetry in the low energy effective theory. For the linear combination

$$U(1) = \sum_x q_x U(1)_x , \quad (3.1)$$

where the respective  $U(1)$  factors originate from the various D-brane stacks, to remain massless the criterion reads [33]

$$\frac{1}{2} \sum_x q_x N_x (\pi_x - \pi'_x) = 0 . \quad (3.2)$$

Here the sum runs over all D-brane stacks  $x$  in the given global setup and  $\pi'_x$  denotes the orientifold image cycle of  $\pi_x$ . In order to avoid later confusions in the discussion of discrete symmetries let us elaborate on constraint (3.2). We introduce a basis of three-cycles  $\{\alpha_i\}$  and  $\{\beta_i\}$  that are even and odd under the orientifold action, respectively, with  $i = 1, \dots, h_{21} + 1$ . The choice of basis is such that  $\alpha_i \cdot \beta_j = \delta_{ij}$  and

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<sup>3</sup>For recent reviews on D-brane model building, see [51–54]. For original work on globally consistent non-supersymmetric intersecting D-branes, see [55–58], and for chiral globally consistent supersymmetric ones, see [11, 59]. For supersymmetric MSSM realizations, see [60–62], and for supersymmetric constructions within type II RCFT's, see [63, 64].

$\alpha_i \cdot \alpha_j = \beta_i \cdot \beta_j = 0$ . Then a three-cycle  $\pi_x$  and its orientifold image  $\pi'_x$  wrapped by a D-brane stack and its image D-brane stack, respectively, can be expanded in terms of this basis

$$\pi_x = \sum_i (m_x^i \alpha_i + n_x^i \beta_i) \quad \pi'_x = \sum_i (m_x^i \alpha_i - n_x^i \beta_i) , \quad (3.3)$$

where  $m_x^i$  and  $n_x^i$  are integer and are usually referred to as wrapping numbers. Using eq. (3.3) the constraint (3.2) takes the form

$$\sum_i \sum_x q_x N_x n_x^i \beta_i = 0 . \quad (3.4)$$

Given that the three-cycles  $\beta_i$  are orthogonal to each other eq. (3.2) reads

$$\sum_x q_x N_x n_x^i = 0 \quad \forall i . \quad (3.5)$$

For a discrete gauge symmetry  $\mathbf{Z}_N$  arising from a linear combination

$$\mathbf{Z}_N = \sum_x k_x U(1)_x \quad (3.6)$$

to survive in the low energy effective field theory it has to satisfy [6]

$$\frac{1}{2} \sum_x k_x N_x (\pi_x - \pi'_x) = 0 \quad \text{mod } N . \quad (3.7)$$

Here we normalize the  $k_x$  to be all integer in order to properly identify the discrete gauge symmetry. Let us clarify the left-hand side of eq. (3.7), which is supposed to indicate that on the left-hand side the basis cycles  $\beta_i$  appear only in multiples of  $N$ . More specifically, using again the expansion of the three-cycles in terms of the basis  $\{\alpha_i\}$  and  $\{\beta_i\}$  the constraint (3.7) reads

$$\sum_x k_x N_x n_x^i = 0 \quad \text{mod } N \quad \forall i , \quad (3.8)$$

In [6, 19] the authors give the constraint for having a discrete symmetry in the form (3.8)<sup>4</sup>. However, for the derivation of stringy bottom-up constraints the cycle constraints (3.2) and (3.7) will turn out to be more appropriate.

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<sup>4</sup>In [6] the actual constraint is given by

$$\sum_x k_x N_x \pi_x \circ \alpha_i = 0 \quad \text{mod } N \quad \forall i$$

which coincides with (3.7) in case  $\alpha_i \circ \beta_j = \delta_{ij}$ , but differs for the cases where  $\alpha_i \circ \beta_j = 2\delta_{ij}$  by a factor of 2. We compensate that by extending the range for the entries  $k_x$  to the set  $[0, 2N - 1]$ . See also, section 4. We thank G. Honecker for pointing out this issue.

Representation	Multiplicity
$\square\square_a$	$\#(\square\square_a) = \frac{1}{2}(\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6})$
$\square_a$	$\#(\square_a) = \frac{1}{2}(\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6})$
$(\square_a, \bar{\square}_b)$	$\#(\square_a, \bar{\square}_b) = \pi_a \circ \pi_b$
$(\square_a, \square_b)$	$\#(\square_a, \square_b) = \pi_a \circ \pi'_b$

**Table 4:** Chiral spectrum of intersection D-branes.

We want to analyse in a bottom-up fashion, consistent with global embedding conditions, i.e. string constraints, analogously to the work of [20–22]<sup>5</sup> what kind of discrete gauge symmetries do appear in (semi-)realistic D-brane compactifications. Given a local configuration of D-brane stacks, where the gauge degrees of freedom are given by open strings localized at a stack of D-branes while the chiral matter is localized at an intersection of two D-brane stacks, the chiral matter content cannot be arbitrary. In contrast, it is subject to severe consistency conditions, namely the tadpole constraint, given by

$$\sum_x N_x (\pi_x + \pi'_x) = 4\pi_{O6} , \quad (3.9)$$

as well as the constraint (3.2) required for the presence of a massless  $U(1)$  in the low energy effective theory. Here  $\pi_{O6}$  in eq. (3.9) denotes the homology class of the orientifold plane.

The equations (3.9) and (3.2) are conditions on the three-cycles the D6-branes wrap, and imply the transformation behaviour of the four-dimensional chiral matter under the D-brane gauge symmetries. More specifically, the chiral matter fields cannot be distributed arbitrarily at the intersections of stacks of D-branes, but they have to obey the above conditions. Those constraints on the transformation behaviour of the matter fields under the D-brane gauge symmetries can be derived by multiplying the equations (3.9) and (3.2) with the three-cycles wrapped by the D6-branes and using table 4.

From the tadpole constraint one obtains [20, 41, 64]

$$\sum_{x \neq a} N_x \left( \#(\square_a, \bar{\square}_x) + \#(\square_a, \square_x) \right) + (N_a - 4)\#(\square_a) + (N_a + 4)\#(\square\square_a) = 0 , \quad (3.10)$$

which is a constraint for each D-brane stack  $a$  of the D-brane setup. Due to the absence of antisymmetric representations for abelian gauge symmetries for a  $U(1)$  stack, for a single D-brane stack, the constraint takes the form

$$\sum_{x \neq a} N_x \left( \#(\square_a, \bar{\square}_x) + \#(\square_a, \square_x) \right) + 5\#(\square\square_a) = 0 \quad \text{mod } 3 . \quad (3.11)$$

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<sup>5</sup>See also, [41, 64, 65].

Note that eq. (3.10) is exactly the anomaly cancellation condition for non-abelian gauge symmetries<sup>6</sup>. However, condition (3.11) has no four-dimensional field theory analogue. It should be stressed that the constraints (3.10) and (3.11) are only necessary constraints. A given tadpole free chiral spectrum arising from a local D-brane configuration does satisfy eq. (3.10) and (3.11). However, a spectrum satisfying eq. (3.10) and (3.11) is not necessarily tadpole free.

The constraints on the transformation behaviour of matter field for having an abelian gauge symmetry in the low energy effective action arising from eq. (3.2) takes the form [20, 41]

$$\begin{aligned} \frac{1}{2} \sum_{x \neq a} q_x N_x \#(\square_a, \bar{\square}_x) - \frac{1}{2} \sum_{x \neq a} q_x N_x \#(\square_a, \square_x) \\ = \frac{q_a N_a}{2(4 - N_a)} \left( \sum_{x \neq a} N_x \left( \#(\square_a, \bar{\square}_x) + \#(\square_a, \square_x) \right) + 8 \#(\square \square_a) \right), \end{aligned} \quad (3.12)$$

where we multiplied equation (3.2) with the homology class of the three-cycles wrapped by the D-brane stack  $a$ . In the derivation of eq. (3.12) we used (3.10) to eliminate the anti-symmetrics. That allows us to display the constraints on the transformation behaviour of the matter fields independently of whether the considered D-brane stack consists of a single or multiple D6-branes. As for the tadpole constraint one has one constraint for each D-brane stack  $a$ . Moreover, the constraints (3.12) do imply the cancellation of abelian cubic anomalies as well as mixed anomalies in four dimensions. However, the constraints imply additional conditions on the transformation behaviour of chiral matter beyond four-dimensional abelian gauge anomaly cancellation. The additional constraints are related to the cancellation of higher dimensional anomalies upon decompactification [9, 12, 13].

Again we would like to stress that condition (3.12) is only a necessary constraint, but not sufficient. This means that not any chiral spectrum arising from a local D-brane configuration satisfying (3.12) for a linear combination of the abelian  $U(1)$  factors does exhibit this abelian gauge symmetry in the low energy effective action.

Let us assume that the MSSM is realized only on a subset of D6-brane stacks, and furthermore that the remaining D6-branes, so called hidden sector D-branes whose presence may be required for global consistency, do not intersect the MSSM D6-branes chirally. For such a scenario the sum over  $x$  in equations (3.10), (3.11) and (3.12) does only contain the visible MSSM D-brane stacks and no knowledge of the hidden sector is necessary.

In the work [20–22] the authors investigated various local D-brane configurations by specifying for each local setup the origin of the chiral MSSM matter, i.e. they

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<sup>6</sup>This statement holds true for all  $SU(N)$  with  $N > 2$ . For  $SU(2)$  field theory does not distinguish between  $\mathbf{2} = \bar{\mathbf{2}}$ , however string theory knows about the  $U(2)$  origin of  $SU(2)$  and thus differentiates between  $\mathbf{2} = \bar{\mathbf{2}}$ .

specify the intersection at which the chiral matter fields do appear, and systematically analyse whether such local configurations do satisfy the severe constraints laid out above. They furthermore required a set of phenomenological bottom-up constraints to ensure compatibility with experimental observations. The latter contain among others the absence of R-parity violating couplings on the perturbative level. We summarize them in appendix A.

Here we want to investigate whether those promising local D-brane configurations found in [20–22], that are consistent with the global consistency conditions, do exhibit discrete gauge symmetries and analyse their phenomenological implications for the low energy effective field theory.

The condition to have a discrete symmetry in a D-brane compactification (3.7) is a constraint on the three-cycles the D6-branes wrap. Just as for the tadpole constraint (3.9) and the masslessness condition (3.2) we will translate this cycle condition into a constraint on the transformation behaviour of the chiral matter by multiplying eq. (3.7) with the homology class of the three-cycles wrapped by the MSSM branes and apply table 4. One obtains

$$\frac{1}{2} \sum_{x \neq a} k_x N_x \left( \#(\square_a, \bar{\square}_x) - \#(\square_a, \square_x) \right) - \frac{k_a N_a}{2} \left( \#(\square\square_a) + \#(\bar{\square}_a) \right) = 0 \pmod{N}, \quad (3.13)$$

which represents a separate constraint for each D-brane stack  $a$ . Note that due to the non-integer prefactor  $\frac{1}{2}$  in equation (3.13) the  $k_x$  do lie in the interval  $(0, 2N - 1)$ . Furthermore, for  $U(1)$  D-brane stacks there are no massless antisymmetrics. In an analogous fashion as for the massless  $U(1)$ 's condition (see eq. (3.12)) we use the tadpole condition (3.10) to eliminate the antisymmetrics that results into

$$\begin{aligned} & \frac{1}{2} \sum_{x \neq a} k_x N_x \#(\square_a, \bar{\square}_x) - \frac{1}{2} \sum_{x \neq a} k_x N_x \#(\square_a, \square_x) \\ & - \frac{k_a N_a}{2(4 - N_a)} \left( \sum_{x \neq a} N_x \left( \#(\square_a, \bar{\square}_x) + \#(\square_a, \square_x) \right) + 8 \#(\square\square_a) \right) = 0 \pmod{N}. \end{aligned} \quad (3.14)$$

One has to be slightly careful in using the tadpole constraint (3.9) to replace the antisymmetrics due to the fact that generically the prefactor is non-integer. This is not an issue for the presence of an abelian gauge symmetry, however can be very crucial for discrete gauge symmetries since the left hand side is not 0 but rather 0 mod  $N$ . One can compensate that by enlarging the interval for the  $k_x$  or by requiring an additional constraint arising from multiplying the homology class of the orientifold

plane with the discrete symmetry constraint (3.7) <sup>7</sup>. This additional constraint reads

$$\sum_a k_a N_a \left( \#(\mathbb{P}_a) - \#(\square_a) \right) = 0 \pmod{N}, \quad (3.15)$$

which after replacing the antisymmetrics in order not to have to distinguish between non-abelian and abelian D-brane stacks takes the form

$$\sum_a \frac{k_a N_a}{4 - N_a} \left( \sum_{x \neq a} N_x \left( \#(\square_a, \bar{\square}_x) + \#(\square_a, \square_x) \right) + 2N_a \#(\square_a) \right) = 0 \pmod{N}. \quad (3.16)$$

Let us mention that the constraints (3.14) and (3.16) do imply the vanishing of the various discrete gauge anomalies, such as  $SU(N) - SU(N) - \mathbf{Z}_N$  or  $G - G - \mathbf{Z}_N$ . However, analogously to the abelian gauge symmetry these string theory constraints are more severe than just four-dimensional discrete gauge anomaly cancellation.

In the subsequent chapter we will investigate the quivers, local D-brane configurations, that were found in [20–22] with respect to discrete symmetries. We will analyse to what extent discrete gauge symmetries do arise and their implications for the low energy action. We will compare those discrete symmetries with the ones found in a pure field theory context [23–25].

## 4. Systematic bottom-up search

In the work [20–22] the authors found various local D-brane configuration, containing up to four D-brane stacks, giving rise to the MSSM spectrum and extensions of it that satisfy the severe top-down constraints arising from string theory, see equations (3.10), (3.11) and (3.12) as well as some minimal set of phenomenological requirements, so called bottom-up constraints. Those contain constraints on R-parity violating couplings, on dangerous dimension 5 proton decay operators as well as on Yukawa textures. In appendix A we summarize those phenomenological bottom-up constraints.

Here we will study those local D-brane configurations, that are consistent with the global consistency conditions, with respect to discrete symmetries. We will analyse what quivers do satisfy the constraints to exhibit discrete symmetries and investigate their implications on the superpotential couplings.

Let us lay out the details of the search. For a chosen  $N$  we check whether a given linear combination of  $U(1)$ 's in terms of the vector  $(k_a, k_b, k_c, k_d)$ , with the  $k_x$ 's being integers, does satisfy the constraints (3.14) and (3.16). Due to the prefactor  $\frac{1}{2}$  in eq. (3.7) we let the  $k_x$  run from 0 to  $2N - 1$ .

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<sup>7</sup>Note that for the abelian gauge symmetry such an additional constraint is not necessary, since one can use the tadpole constraint to replace the homology class of the orientifold plane by all the three-cycles wrapped by the D-brane stacks.

Via a hypercharge shift we can find to any given solution  $(k_a, k_b, k_c, k_d)$  an additional equivalent solutions by adding the hypercharge. Thus  $(k_a + my_a, k_b + my_b, k_c + my_c, k_d + my_d)$  is also a solution to the constraints (3.14) and (3.16) where  $m$  is an integer and the  $y_x$  denote the integer hypercharge embedding coefficients. In order to avoid overcounting we fix the discrete charge of  $Q_L$  for one family to be 0 by choosing  $k_a = k_b$ <sup>8</sup>. Thus we run only over three free integer parameter, namely  $k_a$ ,  $k_c$  and  $k_d$ .

Additionally, we demand that the discrete symmetries allow for the quark and lepton Yukawa couplings in the superpotential, whose presence is crucial for the generation of low energy fermion masses. It turns out that this requirement is very stringent and rules out various discrete symmetries which otherwise satisfy the discrete top-down constraints (3.14) and (3.16).

Finally, we often find solutions for discrete gauge symmetries of higher degree due to the  $\frac{1}{2}$  in (3.14) and (3.16), such as  $\mathbf{Z}_{12}$ , which eventually after determining the matter field charges turn out to be of lower degree from a pure MSSM point of view, since all matter charges have a common divisor. We take those things into account when identifying the discrete symmetries but nevertheless display the linear combinations describing the discrete gauge symmetries in the D-brane language. Therefore, it frequently happens that  $\mathbf{Z}_6$  symmetries contain coefficients that are higher than 12.

In the following we investigate the various promising four-stack quivers, which give rise to the MSSM spectrum (see section 4.1) and the MSSM spectrum with three right-handed neutrinos (see section 4.2). Those promising quivers, that are consistent with the global consistency conditions, were found in a systematic bottom-up search performed in [20–22]. In sections 4.1 and 4.2 we give the details of our findings, specifically we display for each D-brane configuration the possible discrete symmetries and their corresponding vectors  $(k_a, k_b, k_c, k_d)$ . In section 4.3 we present a broad summary of our results.

## 4.1 MSSM realizations

Here we investigate all four stack realizations that give rise to the exact MSSM spectrum, satisfy the severe top-down constraints discussed above and pass the phenomenological constraints displayed in appendix A. Those quivers were found in the systematic bottom-up search performed in [22]. There the authors found D-brane configurations for two different hypercharge embeddings, namely

- $U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b + U(1)_d$
- $U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b$

which we will discuss subsequently.

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<sup>8</sup>In our displayed local D-brane configurations at least one of the left-handed quarks transforms as  $(\square_a, \bar{\square}_b)$  under the D-brane gauge symmetry  $U(3)_a \times U(2)_b$ .

#### 4.1.1 Hypercharge $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$

For this hypercharge embedding there are three different D-brane quivers that give rise to realistic phenomenology. They are displayed in table 5. The first two solutions

#	$Q_L$	$D_R$	$U_R$		$L$	$E_R$		$H_u$	$H_d$	
	$(\square_a, \bar{\square}_b)$	$(\bar{\square}_a, \bar{\square}_c)$	$(\bar{\square}_a, \bar{\square}_d)$	$\bar{\square}_a$	$(\square_b, \bar{\square}_c)$	$(\square_c, \square_d)$	$\bar{\square}_b$	$(\square_b, \square_d)$	$(\square_b, \square_c)$	$(\bar{\square}_b, \bar{\square}_d)$
1	3	3	3	0	3	1	2	1	1	0
2	3	3	3	0	3	0	3	1	0	1
3	3	3	0	3	3	0	3	1	0	1

**Table 5:** MSSM spectrum for setups with  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$ .

of table 5 exhibit a discrete  $\mathbf{Z}_3$  symmetry which can be identified with  $L_3 R_3$ , i.e. Baryon triality. The linear combination is given by

$$L_3 R_3 = U(1)_a + U(1)_b + 3U(1)_c + U(1)_d \quad (4.1)$$

and satisfies the constraints (3.14) and (3.16). Thus both models may exhibit a Baryon triality.

The third solution of table 5 may even have an additional massless  $U(1)$  given by

$$U^{add}(1) = U(1)_d . \quad (4.2)$$

However, it should be absent since otherwise it would spoil the presence of desired Yukawa couplings. Even any discrete subgroup of  $U^{add}(1)$  is forbidden, since it does not allow any of the superpotential terms  $Q_L H_u U_R$ ,  $Q_L H_d D_R$  nor  $L H_d E_R$ . Apart from this additional undesired  $U^{add}(1)$  and its potential undesired discrete subgroups this local D-brane setup does not exhibit any further discrete symmetries.

#### 4.1.2 Hypercharge $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$

#	$Q_L$	$D_R$	$U_R$	$L$	$E_R$	$H_u$	$H_d$
	$(\square_a, \bar{\square}_b)$	$(\bar{\square}_a, \bar{\square}_d)$	$\bar{\square}_a$	$(\square_b, \bar{\square}_d)$	$\bar{\square}_b$	$(\bar{\square}_b, \bar{\square}_c)$	$(\square_b, \square_c)$
1	3	3	3	3	3	1	1

**Table 6:** MSSM spectrum for setups with  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$ .

The solution # 1 displayed in table 6 may exhibit an additional  $U^{add}(1)$  that is given by

$$U^{add}(1) = U(1)_c \quad (4.3)$$

satisfying the necessary constraints (3.12). However, such an abelian gauge symmetry and any discrete subgroup of it should be absent, since otherwise the desired Yukawa couplings  $Q_L H_u U_R$ ,  $Q_L H_d D_R$  and  $L H_d E_R$  would be forbidden.

Apart from this additional  $U^{add}(1)$  we find no vector  $(k_a, k_b, k_c, k_d)$  that satisfies the discrete anomaly constraints (3.14) and (3.16). Thus in such a configuration one cannot find any discrete gauge symmetry, which may help to explain the absence of various undesired superpotential terms and the absence of R-parity violating terms or dimension 5 proton decay operators is rather accidental.

## 4.2 MSSM + three right-handed neutrino realizations

Here we analyse all four stack realizations exhibiting the MSSM spectrum plus three right-handed neutrinos, satisfying the severe top-down constraints and allowing for an acceptable phenomenology. Those quivers were found in a systematic search performed in [20,21], where the authors found only four solutions for the hypercharge embeddings listed below

- $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{3}{2}U(1)_d$
- $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$
- $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$
- $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$ .

They will be analysed with respect to discrete gauge symmetries in the following.

### 4.2.1 Hypercharge $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{3}{2}U(1)_d$

#	$Q_L$	$D_R$	$U_R$	$L$	$E_R$	$N_R$	$H_u$	$H_d$
	$(\square_a, \bar{\square}_b)$	$\bar{\square}_a$	$(\bar{\square}_a, \bar{\square}_c)$	$(\square_b, \bar{\square}_c)$	$(\bar{\square}_c, \bar{\square}_d)$	$\bar{\square}_b$	$(\square_b, \square_c)$	$(\bar{\square}_b, \bar{\square}_c)$
1	3	3	3	3	3	3	1	1

**Table 7:** MSSM + 3  $N_R$  spectrum for setups with  $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{3}{2}U(1)_d$ .

In the model # 1 of table 7 the matter fields transform in such a way that there may be an additional  $U^{add}(1)$  given by the linear combination

$$U^{add}(1) = U(1)_a + U(1)_b + U(1)_c - 3U(1)_d, \quad (4.4)$$

which allows for all the desired Yukawa couplings, and together with the hypercharge gives the B-L symmetry:  $U(1)_{B-L} = 2U(1)_Y - \frac{1}{2}U^{add}(1)$ . Clearly any discrete subgroup of the gauge symmetry  $U^{add}(1)$  will satisfy the constraints (3.14) and (3.16). The  $\mathbf{Z}_2$ ,  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  discrete subgroups of  $U^{add}(1)$  correspond to  $R_2$ ,  $R_3$  and  $R_6$ , respectively. Beyond the discrete gauge subgroups of  $U^{add}(1)$  the local setup does not exhibit any additional discrete gauge symmetries.

### 4.2.2 Hypercharge $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$

#	$Q_L$	$D_R$	$U_R$	$L$	$E_R$	$N_R$								$H_u$	$H_d$
	$(\square_a, \square_b)$	$(\square_a, \square_d)$	$\square_b$	$(\square_b, \square_d)$	$\square_b$	$(\square_c, \square_d)$	$(\square_c, \square_t)$	$(\square_c, \square_l)$	$\square_c$	$\square_c$	$\square_d$	$\square_d$	$(\square_b, \square_c)$	$(\square_b, \square_c)$	
1	3	3	3	3	3	0	0	0	0	0	0	3	1	1	
2	3	3	3	3	3	0	0	0	0	0	3	0	1	1	
3	3	3	3	3	3	0	0	3	0	0	0	0	1	1	
4	3	3	3	3	3	0	3	0	0	0	0	0	1	1	
5	3	3	3	3	3	1	1	0	0	0	0	1	1	1	
6	3	3	3	3	3	3	0	0	0	0	0	0	1	1	
7	3	3	3	3	3	0	0	1	0	1	0	1	1	1	
8	3	3	3	3	3	0	1	0	0	1	1	0	1	1	
9	3	3	3	3	3	0	0	0	0	3	0	0	1	1	
10	3	3	3	3	3	0	1	1	1	0	0	0	1	1	
11	3	3	3	3	3	1	0	0	1	0	1	0	1	1	
12	3	3	3	3	3	0	0	0	3	0	0	0	1	1	

**Table 8:** MSSM + 3  $N_R$  spectrum for setups with  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$ .

The solutions # 1, # 3 and # 5 of table 8 may exhibit an additional  $U(1)$ , satisfying the constraints (3.12). However, it should be noted that those  $U(1)$ 's cannot be realized in a realistic compactification, since their presence would forbid some of the desired Yukawa couplings  $Q_L H_u U_R$ ,  $Q_L H_d D_R$  or  $L H_d E_R$ . Not even a discrete subgroup of those abelian gauge symmetries is allowed since for any discrete subgroup the absence of the desired Yukawa couplings holds true.

The solution # 2 may even exhibit two independent abelian gauge symmetries, namely

$$U_1^{add}(1) = U(1)_c \quad \text{and} \quad U_2^{add}(1) = U(1)_b - 2U(1)_d \quad (4.5)$$

Only the linear combination  $U^f(1) = -3U(1)_Y + U_1^{add}(1) - \frac{1}{2}U_2^{add}(1)$  should be indeed realized, since otherwise various desired Yukawa couplings would be not allowed. This implies the presence of a B-L symmetry that is given by  $U(1)_{B-L} = 2U(1)_Y + \frac{1}{2}U^f(1)$ . Again the constraints (3.12) are only necessary constraints and not sufficient, but clearly any subgroup of  $U^f(1)$  does satisfy the discrete anomaly constraints (3.14) and (3.16). Thus even though  $U^f(1)$  may not be realized in a concrete compactification it may well be that a discrete subgroup survives. Among those subgroups rank the  $\mathbf{Z}_2$ ,  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  discrete symmetries  $R_2$ ,  $R_3$  and  $R_6$ .

In addition to the above mentioned observations we find for all solutions apart for solutions # 4, # 5, # 8 and # 10 the discrete symmetry  $R_2$  realized, where the matter field charges are given by

$$R_2 = U(1)_a + U(1)_b + U(1)_c + U(1)_d. \quad (4.6)$$

As can be seen from table 2 this matter parity  $R_2$  forbids the presence of R-parity violating couplings. Beyond matter parity  $R_2$  none of the twelve setups exhibits

any additional discrete gauge symmetries, apart from solution # 2 which allows for discrete subgroups of  $U^f(1)$ .

#### 4.2.3 Hypercharge $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$

#	$Q_L$	$D_R$	$U_R$		$L$	$E_R$		$N_R$		$H_u$	$H_d$	
	$(\square_a, \bar{\square}_b)$	$(\bar{\square}_a, \bar{\square}_c)$	$(\bar{\square}_a, \bar{\square}_d)$	$\bar{\square}_a$	$(\square_b, \bar{\square}_c)$	$(\square_c, \square_d)$	$\bar{\square}_b$	$\square_c$	$\bar{\square}_c$	$(\square_b, \square_d)$	$(\square_b, \square_c)$	$(\bar{\square}_b, \bar{\square}_d)$
1	3	3	3	0	3	1	2	0	3	1	1	0
2	3	3	3	0	3	1	2	3	0	1	1	0
3	3	3	0	3	3	0	3	0	3	1	0	1
4	3	3	0	3	3	0	3	3	0	1	0	1

**Table 9:** MSSM + 3  $N_R$  spectrum for setups with  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$ .

The solution # 1 of table 9 satisfies all constraints for matter parity, Baryon triality and hence also for Proton hexality. Matter parity  $R_2$  and Baryon triality  $L_3 R_3$  are given by

$$R_2 = U(1)_a + U(1)_b + U(1)_c + 5U(1)_d \quad (4.7)$$

$$L_3 R_3 = U(1)_a + U(1)_b + 3U(1)_c + U(1)_d . \quad (4.8)$$

Proton hexality takes the form

$$L_6^2 R_6^5 = U(1)_a + U(1)_b + 9U(1)_c + 13U(1)_d \quad (4.9)$$

and does prevent the presence of R-parity violating couplings as well as the presence of dangerous dimension 5 proton decay operators, and at the same time allows for a  $\mu$ -term as well as a Weinberg operator.

The second solution of table 9 exhibits a massless  $U(1)$  of the form

$$U^{add}(1) = U(1)_a + U(1)_b + U(1)_c - 3U(1)_d \quad (4.10)$$

which does not forbid any desired Yukawa couplings whereas the B-L symmetry takes the form  $U(1)_{B-L} = 2U(1)_Y + \frac{1}{2}U^{add}(1)$ . As before any discrete subgroup satisfies the constraints for the discrete symmetry (3.14) and (3.16). For instance the  $\mathbf{Z}_2$  subgroup of  $U^{add}$  can be interpreted as matter parity. Moreover, one finds all four different discrete  $\mathbf{Z}_3$  symmetries found in the MSSM using the pure field theoretical ansatz. They are given by the following linear combinations

$$L_3 R_3^2 = 2U(1)_c + 4U(1)_d \quad (4.11)$$

$$L_3 = U(1)_a + U(1)_b + 5U(1)_c + 5U(1)_d \quad (4.12)$$

$$R_3 = U(1)_a + U(1)_b + U(1)_c + 3U(1)_d \quad (4.13)$$

$$L_3 R_3 = U(1)_a + U(1)_b + 3U(1)_c + U(1)_d , \quad (4.14)$$

where  $R_3$  originates from  $U^{add}(1)$ . Only Baryon triality  $L_3 R_3$  allows for the presence of a Weinberg operator. Thus in presence of the other discrete symmetries it is challenging to find a mechanism to generate neutrino masses. Finally, the setup also satisfies the constraints to exhibit all of the  $\mathbf{Z}_6$  symmetries, i.e.

$$L_6^2 R_6 = 3U(1)_a + 3U(1)_b + 19U(1)_c + 23U(1)_d \quad (4.15)$$

$$L_6^2 R_6^3 = U(1)_a + U(1)_b + 17U(1)_c + 5U(1)_d \quad (4.16)$$

$$L_6^2 R_6^5 = U(1)_a + U(1)_b + 9U(1)_c + 13U(1)_d \quad (4.17)$$

$$R_6 = U(1)_a + U(1)_b + U(1)_c + 21U(1)_d, \quad (4.18)$$

where  $R_6$  originates from  $U^{add}(1)$ . In contrast to the solution # 1 here the proton hexality may be realized as a subgroup of a larger symmetry, namely a combination of the abelian gauge symmetry  $U^{add}(1)$  and the discrete symmetry  $L_3 R_3$ . In a concrete realization of this setup the  $B \wedge F$  couplings may break the  $U^{add}(1)$  down to matter parity  $R_2$  and thus only Proton hexality survives in the low energy limit. In case a larger symmetry survives the  $B \wedge F$  couplings one needs a dynamical mechanism for the larger symmetry to break down to Proton hexality since otherwise the generation of a Weinberg operator and  $\mu$ -term is not allowed.

The solution # 3 of table 9 may exhibit an additional  $U^{add}(1) = U(1)_d$  which potentially remains massless, i.e. it satisfies the constraints (3.12). However, the presence of such an abelian gauge symmetry would spoil the model, since it would forbid various desired Yukawa couplings. Even worse there exists no discrete subgroup of the abelian gauge symmetry  $U^{add}(1)$  that would allow the desired Yukawa couplings. Thus in a concrete realization it must be absent. The local D-brane configuration however does allow for a discrete  $\mathbf{Z}_2$  that allows all desired Yukawa couplings, the matter parity  $R_2$ , given by

$$R_2 = U(1)_a + U(1)_b + U(1)_c + U(1)_d \quad (4.19)$$

which forbids all R-parity violating couplings.

The solution # 4 of table 9 may exhibit two additional  $U(1)$ 's given by

$$U_1^{add}(1) = U(1)_b - 2U(1)_c \quad \text{and} \quad U_2^{add}(1) = U(1)_d \quad (4.20)$$

where the latter cannot survive as a gauge symmetry since it would forbid all desired Yukawa couplings. On the other hand the abelian gauge symmetry  $U_1^{add}(1)$  does allow all superpotential terms. The B-L symmetry is given by  $U_{B-L}(1) = \frac{1}{2}U_Y(1) - \frac{1}{4}U_1^{add}(1)$  in terms of the hypercharge and the additional  $U_1^{add}(1)$ . One finds for this configuration that the discrete subgroup of the two abelian gauge symmetries  $U(1)_Y$  and  $U_1^{add}(1)$  do give rise to matter parity  $R_2$ , to the  $\mathbf{Z}_3$  symmetry  $R_3$  and to the  $\mathbf{Z}_6$  symmetry  $R_6$ . These discrete gauge symmetries are realized as the following linear

combinations

$$R_2 = U(1)_a + U(1)_b + U(1)_c + U(1)_d \quad (4.21)$$

$$R_3 = U(1)_a + U(1)_b + U(1)_c \quad (4.22)$$

$$R_6 = U(1)_a + U(1)_b + U(1)_c + 9U(1)_d, \quad (4.23)$$

While  $R_2$  forbids all R-parity violating couplings in this local D-brane configuration the absence of dimension 5 proton decay operators is rather accidental and does not originate from a discrete gauge symmetry.

#### 4.2.4 Hypercharge $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$

#	$Q_L$	$D_R$		$U_R$		$L$		$E_R$		$N_R$			$H_u$		$H_d$	
	$(\square_a, \bar{\square}_b)$	$(\bar{\square}_a, \square_c)$	$(\bar{\square}_a, \bar{\square}_d)$	$(\bar{\square}_a, \bar{\square}_c)$	$(\bar{\square}_a, \square_d)$	$(\square_b, \bar{\square}_c)$	$(\square_b, \square_d)$	$(\square_c, \bar{\square}_d)$	$\square_c, \square_d$	$\bar{\square}_b$	$(\square_c, \square_d)$	$(\bar{\square}_c, \bar{\square}_d)$	$(\square_b, \square_c)$	$(\square_b, \bar{\square}_d)$	$(\square_b, \bar{\square}_c)$	$(\bar{\square}_b, \bar{\square}_c)$
1	3	3	0	0	3	0	3	0	0	3	2	0	1	0	1	0
2	3	3	0	0	3	0	3	1	0	2	2	1	0	0	1	0
3	3	3	0	2	1	0	3	2	1	0	2	1	0	0	1	0
4	3	3	0	2	1	0	3	0	2	1	2	1	0	0	1	0
5	3	3	0	3	0	0	3	2	1	0	2	0	1	1	0	0
6	3	3	0	3	0	0	3	0	2	1	2	0	1	1	0	0
7	3	3	0	3	0	0	3	1	2	0	2	1	0	1	0	0
8	3	0	3	0	3	3	0	2	0	1	3	0	0	0	1	0
9	3	0	3	0	3	3	0	0	1	2	3	0	0	0	1	0
10	3	0	3	1	2	3	0	3	0	0	3	0	0	1	0	0
11	3	0	3	1	2	3	0	1	1	1	3	0	0	1	0	0
12	3	0	3	3	0	3	0	0	3	0	3	0	0	1	0	0

**Table 10:** MSSM + 3  $N_R$  spectrum for setups with  $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$ .

All solutions of table 10 apart from # 3, # 4, # 10 and # 11 exhibit the discrete  $\mathbf{Z}_3$  symmetry  $L_3 R_3$ , i.e. Baryon triality. For all these solutions the Baryon triality is given by the same linear combination, namely

$$L_3 R_3 = 2U(1)_c + 4U(1)_d. \quad (4.24)$$

Apart from those discrete symmetry  $L_3 R_3$  only the solutions # 1 and # 12 may contain more discrete symmetries. They both may exhibit an additional massless  $U^{add}(1)$ <sup>9</sup> which allows all desired Yukawa couplings. Moreover, it turns out that both solutions do give rise to matter parity  $R_2$  and all possible  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  symmetries.

For solution # 1 of table 10 the matter parity takes the form

$$R_2 = U(1)_a + U(1)_b + 3U(1)_c + 7U(1)_d \quad (4.25)$$

---

<sup>9</sup>For solution # 1 the additional  $U(1)$  takes the form  $U^{add}(1) = -U(1)_b - 2U(1)_d$  while for solution # 12 it is given by  $U^{add}(1) = U(1)_b - 2U(1)_c$ . The form of the B-L symmetry is given by  $U_{B-L}(1) = -U_Y(1) + \frac{1}{2}U^{add}(1)$  and  $U_{B-L}(1) = -U_Y(1) - \frac{1}{2}U^{add}(1)$ , respectively.

while the  $\mathbf{Z}_3$  symmetries are given by

$$R_3 = U(1)_a + U(1)_b + 3U(1)_c + 5U(1)_d \quad (4.26)$$

$$L_3 = U(1)_a + U(1)_b + U(1)_c + U(1)_d \quad (4.27)$$

$$L_3 R_3 = 2U(1)_c + 4U(1)_d \quad (4.28)$$

$$L_3 R_3^2 = U(1)_a + U(1)_b + 5U(1)_c + 3U(1)_d . \quad (4.29)$$

The  $\mathbf{Z}_6$  symmetries read

$$R_6 = U(1)_a + U(1)_b + 3U(1)_c + 23U(1)_d \quad (4.30)$$

$$L_6^2 R_6 = U(1)_a + U(1)_b + 11U(1)_c + 15U(1)_d \quad (4.31)$$

$$L_6^2 R_6^3 = U(1)_a + U(1)_b + 19U(1)_c + 7U(1)_d \quad (4.32)$$

$$L_6^2 R_6^5 = 3U(1)_a + 3U(1)_b + U(1)_c + 5U(1)_d . \quad (4.33)$$

For solution # 12 of table 10 the matter parity is given by the linear combination

$$R_2 = U(1)_a + U(1)_b + U(1)_c + 5U(1)_d \quad (4.34)$$

The  $\mathbf{Z}_3$  symmetries take the form

$$R_3 = U(1)_a + U(1)_b + U(1)_c + 3U(1)_d \quad (4.35)$$

$$L_3 = U(1)_a + U(1)_b + 5U(1)_c + 5U(1)_d \quad (4.36)$$

$$L_3 R_3 = 2U(1)_c + 4U(1)_d \quad (4.37)$$

$$L_3 R_3^2 = U(1)_a + U(1)_b + 3U(1)_c + U(1)_d . \quad (4.38)$$

and the  $\mathbf{Z}_6$  symmetries read

$$R_6 = U(1)_a + U(1)_b + U(1)_c + 21U(1)_d \quad (4.39)$$

$$L_6^2 R_6 = U(1)_a + U(1)_b + 9U(1)_c + 13U(1)_d \quad (4.40)$$

$$L_6^2 R_6^3 = U(1)_a + U(1)_b + 17U(1)_c + 5U(1)_d \quad (4.41)$$

$$L_6^2 R_6^5 = 3U(1)_a + 3U(1)_b + 11U(1)_c + 7U(1)_d . \quad (4.42)$$

Beyond the discrete  $\mathbf{Z}_2$ ,  $\mathbf{Z}_3$ ,  $\mathbf{Z}_6$  gauge symmetries as well as the subgroups of the additional  $U^{add}(1)$  both solutions, # 1 and # 12, do not possess any further family dependent discrete gauge symmetries.

Again in contrast to the solution # 1 of table 9 the Proton hexality in solution # 1 and # 12 may be realized as a subgroup of a larger symmetry, namely a combination of the abelian gauge symmetry  $U^{add}(1)$  and the discrete symmetry, matter parity  $R_2$ . In a concrete realization of this setup the  $B \wedge F$  couplings may break the  $U^{add}(1)$  down to matter parity  $R_2$  and thus only Proton hexality survives in the low energy limit. However, in case of a larger symmetry surviving the Green-Schwarz mechanism one needs a dynamical mechanism for the larger symmetry to break down to Proton hexality. Otherwise one would face severe difficulties in the generation of a Weinberg operator and  $\mu$ -term, since the larger symmetry does forbid them.

#### 4.2.5 $SU(2)$ realized as $Sp(2)$ with $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$

Since  $Sp(2)$  is isomorphic to  $SU(2)$  we can realize the  $SU(2)$  of the MSSM as a D-brane stack wrapping an orientifold invariant cycle. A D-brane stack that wraps an orientifold invariant cycle, thus giving rise to a  $Sp(2)$  symmetry does not contain a  $U(1)$  gauge factor that can contribute to the hypercharge or a discrete symmetry. Moreover, it should be noted that the tadpole constraint for the  $Sp(2)$  stack does not give any constraints on the transformation behaviour of the chiral matter fields. While the last statement seems to suggest that one finds multiple local D-brane configurations that satisfy the severe top-down constraints and exhibiting a (semi-) realistic phenomenology the systematic search performed in [20] finds only one configuration displayed in table 11.

#	$Q_L$	$D_R$	$U_R$	$L$	$E_R$	$N_R$	$H_u$	$H_d$
	$(\square_a, \square_b)$	$(\overline{\square}_a, \square_c)$	$(\overline{\square}_a, \overline{\square}_c)$	$(\square_b, \square_d)$	$(\square_c, \overline{\square}_d)$	$(\overline{\square}_c, \overline{\square}_d)$	$(\square_b, \square_c)$	$(\square_b, \overline{\square}_c)$
1	3	3	3	3	3	3	1	1

**Table 11:** MSSM + 3  $N_R$  spectrum for setups with  $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$ .

This model may exhibit a massless  $U^{add}(1)$  which is given by

$$U^{add}(1) = U(1)_c . \quad (4.43)$$

and contains the discrete symmetries  $R_2$ ,  $R_3$  and  $R_6$ . The abelian gauge symmetry  $U^{add}(1)$  can be combined with  $U(1)_Y$  such that it gives the  $B - L$  symmetry

$$U(1)_{B-L} = 2U(1)_Y - U^{add}(1) . \quad (4.44)$$

In addition the configuration displayed in table 11 may exhibit the  $\mathbf{Z}_3$  symmetries  $L_3$ ,  $L_3R_3$  and  $L_3R_3^2$  given by the linear combination

$$L_3 = 2U(1)_d \quad (4.45)$$

$$L_3R_3 = 2U(1)_c + 4U(1)_d \quad (4.46)$$

$$L_3R_3^2 = U(1)_c + 4U(1)_d \quad (4.47)$$

as well as the  $\mathbf{Z}_6$  discrete symmetries  $L_6^2R_6$ ,  $L_6^2R_6^3$  and  $L_6^2R_6^5$  which take the form

$$L_6^2R_6 = U(1)_c + 4U(1)_d \quad (4.48)$$

$$L_6^2R_6^3 = 3U(1)_c + 4U(1)_d \quad (4.49)$$

$$L_6^2R_6^5 = U(1)_c + 8U(1)_d . \quad (4.50)$$

As before Proton hexality may appear as a subgroup of a larger symmetry depending on the details of the Stückelberg-type couplings in a concrete realization. In that case

one needs a dynamical mechanism to further break the larger symmetry to Proton hexality in order to allow for a  $\mu$ -term and a Weinberg operator.

Beyond those discrete gauge symmetries the local D-brane configuration does not exhibit any additional discrete gauge symmetry. In particular the D-brane setup does not possess any family dependent discrete gauge symmetries.

### 4.3 Summary of the results

Let us give a brief summary of the results of the systematic bottom-up search performed above. The first thing to note is that we do not find in any of the intriguing four stack quivers family dependent discrete gauge symmetries that allow for the desired Yukawa couplings  $Q_L H_u U_R$ ,  $Q_L H_d D_R$  and  $L H_d E_R$ . This is somewhat not expected since specifically the leptons in those D-brane configurations do arise from different intersections of D-brane stacks, and thus transform differently under the anomalous  $U(1)$  factors. Nevertheless after determining the discrete charge of all matter fields all generations do have the same charge, even though their D-brane origin is significantly different.

The second observation is that we do not find any discrete  $\mathbf{Z}_9$  and  $\mathbf{Z}_{18}$  symmetries for the local MSSM D-brane configurations, which can appear in the pure field theoretical approach. This is due to the more constraining conditions for the appearance of discrete symmetries in D-brane compactifications.

Table 12 displays for each quiver the potential appearing discrete symmetries. It shows that matter parity  $R_2$  is favoured for the hypercharge embeddings

$$U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b \quad \text{and} \quad (4.51)$$

$$U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d \quad (4.52)$$

which appears for almost all D-brane setups with these hypercharge embeddings. For the hypercharge embedding in eq. (4.52)  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$  there is only one configuration out of 12 that allows for a  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  discrete symmetry. On the other hand for the hypercharge embedding  $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$  we do find for each realization a  $\mathbf{Z}_3$  symmetry, but only in two cases it is Baryon triality. Those local D-brane configurations also allow for Proton hexality.

For the Madrid embedding

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d .$$

almost all realizations have the potential to exhibit Baryon triality. However, the presence of matter parity is highly suppressed. Only for two setups we also find matter parity realized. Hence, those quivers pass the constraints to exhibit Proton hexality.

Spectrum	Hypercharge	Table	#	$R_2$	$L_3 R_3$	$R_3$	$L_3$	$L_3 R_3^2$	$L_6^2 R_6^5$	$R_6$	$L_6^2 R_6^3$	$L_6^2 R_6$
MSSM	$(-\frac{1}{3}, -\frac{1}{2}, 0, 1)$	5	1		✓							
			2		✓							
			3									
MSSM + 3 $N_R$	$(-\frac{1}{3}, -\frac{1}{2}, 0, 1)$	6	1									
	$(\frac{1}{6}, 0, \frac{1}{2}, -\frac{3}{2})$	7	1	✓		✓				✓		
	$(-\frac{1}{3}, -\frac{1}{2}, 0, 0)$	8	1	✓								
			2	✓		✓				✓		
			3	✓								
			4									
			5									
			6	✓								
			7	✓								
			8									
			9	✓								
			10									
			11	✓								
			12	✓								
	$(-\frac{1}{3}, -\frac{1}{2}, 0, 1)$	9	1	✓	✓				✓			
			2	✓	✓	✓	✓	✓	✓	✓	✓	✓
			3	✓								
			4	✓		✓				✓		
	$(\frac{1}{6}, 0, \frac{1}{2}, -\frac{1}{2})$	10	1	✓	✓	✓	✓	✓	✓	✓	✓	✓
			2		✓							
			3									
			4									
			5		✓							
			6		✓							
			7		✓							
			8		✓							
			9		✓							
			10									
			11									
			12	✓	✓	✓	✓	✓	✓	✓	✓	✓
	$(\frac{1}{6}, 0, \frac{1}{2}, -\frac{1}{2})$	11	1	✓	✓	✓	✓	✓	✓	✓	✓	✓

**Table 12:** The table summarizes our findings on the search of discrete gauge symmetries in promising local D-brane setups. The symbol ✓ denotes the potential presence of a discrete gauge symmetry for the respective local D-brane setup. Matter parity is given by  $R_2$ , Baryon triality by  $L_3 R_3$  and Proton hexality by  $L_6^2 R_6^5$ .

Summarizing we find only five setups that have the potential to exhibit Proton hexality, which is a particular intriguing discrete symmetry since it forbids all R-parity violating terms as well as all dangerous dimension 5 proton decay operators. This suggests that the presence of Proton hexality in D-brane compactifications is rather suppressed.

Finally, one observes a similar pattern as in the field theoretical approach, namely that the presence of discrete  $\mathbf{Z}_6$  symmetries is tied to the presence of  $\mathbf{Z}_2$  and  $\mathbf{Z}_3$  symmetries. We find the same relations as in pure field theory

$$R_2 \times L_3 R_3 \cong L_6^2 R_6^5 \quad (4.53)$$

$$R_2 \times R_3 \cong R_6 \quad (4.54)$$

$$R_2 \times L_3 \cong L_6^2 R_6^3 \quad (4.55)$$

$$R_2 \times L_3 R_3^2 \cong L_6^2 R_6 \ . \quad (4.56)$$

Thus, the presence of  $R_2$  along with a discrete  $\mathbf{Z}_3$  symmetry implies the presence of a  $\mathbf{Z}_6$  symmetry.

## 5. Conclusions

We study the presence of discrete gauge symmetries in D-brane compactifications. We translate the conditions for the presence of a discrete gauge symmetry in D-brane compactifications laid out in [6] into constraints on the transformation behaviour of the chiral matter fields. This allows for a bottom-up search, a search that does not require the knowledge of any features of the compactification manifold, for local D-brane configurations with respect to discrete gauge symmetries.

After establishing those constraints on the transformation behaviour of the chiral matter fields we perform a systematic search for discrete gauge symmetries within a class of promising local D-brane quivers based on four stacks of D-branes. Those local configurations, that are consistent with the global consistency conditions, were found in [20–22] and exhibit the exact MSSM spectrum or the exact MSSM spectrum plus three right-handed neutrinos. Within this class of intriguing four stack quivers there is no quiver that allows for a family dependent discrete gauge symmetry. Moreover, none of the local MSSM D-brane configurations exhibits a discrete  $\mathbf{Z}_9$  and  $\mathbf{Z}_{18}$  gauge symmetry, which, on the other hand, were found in [23] using a pure field theoretical approach. This confirms one of our earlier findings that the constraints on the transformation behaviour of the chiral matter fields for having a discrete gauge symmetry in D-brane compactifications goes beyond the four-dimensional discrete gauge anomaly conditions used in [23].

Our search reveals that all  $\mathbf{Z}_2$ ,  $\mathbf{Z}_3$  and  $\mathbf{Z}_6$  discrete gauge symmetries found in [23] can be also realized in the local D-brane configurations. We find that the realization of discrete symmetries depends on the hypercharge embedding of the D-brane configuration. For instance while the Madrid embedding favours Baryon triality it disfavors matter parity. The presence of Proton hexality, i.e. the simultaneous presence of matter parity and Baryon triality, is rather suppressed and only realized for five of the intriguing four D-brane-stack quivers. In those quivers the absence of

R-parity and disastrous dimension 5 proton decay operators is not accidental, but can be explained by the presence of a discrete gauge symmetry.

It would be interesting to extend this analysis to local semi-realistic D-brane configurations with more than 4 D-brane stacks. Specifically, it would be interesting to see whether one can find family dependent discrete gauge symmetries in those realizations. Furthermore, another intriguing avenue is to extend the analysis to the NMSSM [40] and GUT realizations of the MSSM [41] as well as extending it to local D-brane configurations with additional exotics [42].

Finally, we would like to comment on the limits of the bottom-up approach applied here. The discrete gauge symmetries considered here purely originate from the anomalous  $U(1)$  factors carried by each D-brane stack. In addition there may be abelian or even non-abelian gauge factors arising from isometries of the compactification manifold which can lead to abelian and non-abelian discrete gauge symmetries in the low energy effective action [47]. The consideration of discrete symmetries originating from isometries, requires the specification of the properties of the compactification and thus goes beyond the scope of this work.

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## A. Bottom-up constraints

In this appendix we display all the bottom-up constraints which were imposed in the search of realistic local D-brane configurations. Apart from the top-down constraints (3.10), (3.11) and (3.12) that the spectrum has to satisfy we furthermore require a few phenomenological constraints to be satisfied

- The MSSM superpotential couplings

$$Q_L H_u U_R \quad Q_L H_d D_R \quad L H_d E_R \quad (A.1)$$

are either realized perturbatively or in case they violate global  $U(1)$  selection rules and thus are perturbatively forbidden they will be induced by D-instantons, such that all three families of quarks and charged leptons acquire masses.

- We require that the D-brane quiver exhibits a mechanism which accounts for the neutrino masses [16, 17, 48, 49, 66–68].
- We demand the absence of the R-parity violating couplings

$$D_R D_R U_R \quad L L E_R \quad Q_L L D_R \quad L H_u \quad (A.2)$$

on the perturbative level and furthermore, require that they are not generated by a D-instanton whose presence is required to generate a perturbatively forbidden, but desired, MSSM superpotential couplings.

- We demand that none of the D-instantons required to generate desired Yukawa couplings does induce a tadpole  $N_R$ .
- We rule out setups which lead to a large family mixing in the quark Yukawa couplings [20, 35, 36, 69, 70].
- We demand the absence of the dangerous dimension 5 proton decay operators

$$U_R U_R D_R E_R \quad \text{and} \quad Q_L Q_L Q_L L \quad (A.3)$$

on the perturbative level and additionally require that they are not generated by a D-instanton whose presence is required to generate a perturbatively forbidden, but desired, MSSM superpotential couplings.

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